Reg. No. | | | | | | | | |

G. VENKATASWAMY NAIDU COLLEGE (AUTONOMOUS), KOVILPATTI – 628 502.

UG DEGREE END SEMESTER EXAMINATIONS - NOVEMBER 2024.

(For those admitted in June 2023 and later)

PROGRAMME AND BRANCH: B.Sc., MATHEMATICS

SEM	CATEGORY	COMPONENT	COURSE CODE	COURSE TITLE
III	PART - III	CORE-5	U23MA305	VECTOR CALCULUS AND ITS APPLICATIONS

Date & Session: 09.11.2024 / AN **Time : 3 hours** **Maximum: 75 Marks**

Course Outcome	Bloom's K-level	Q. No.	**SECTION – A (10 X 1 = 10 Marks)** Answer **ALL** Questions.
CO1	K1	1.	If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$, then $\nabla.\bar{r} =$ _____. a) 0 b) 3 c) -3 d) 1
CO1	K2	2.	The vector field $\bar{F}$ is solenoidal if _____ a) $\nabla.\bar{F} = 0$ b) $\nabla \times \bar{F} = 0$ c) $\nabla^2\bar{F} = 0$ d) $\nabla.\bar{F} = \bar{0}$
CO2	K1	3.	Divergence of the curl of a twice differentiable continuous function is ____. a) 1 b) 0 c) ∞ d) $-\infty$
CO2	K2	4.	Curl curl A =______. a) $\nabla \times (\nabla \times A)$ b) $\Delta \times (\Delta \times A)$ c) $\nabla.(\nabla.A)$ d) $\Delta.(\Delta.A)$
CO3	K1	5.	A field $\bar{F}$ is said to be conservative if ______. a) $\nabla.\bar{F} = 0$ b) $\nabla \times \bar{F} = 0$ c) $\nabla^2\bar{F} = 0$ d) $\nabla.\bar{F} = \bar{0}$
CO3	K2	6.	The value of $\oint dl$ along a circle of radius 2 unit is ____. a) 0 b) 2π c) 4π d) 8π
CO4	K1	7.	Which type of surface is circular cylinder? a) topological surface b) implicit surface c) Algebraic and differential surface d) fractal surface
CO4	K2	8.	The parabolic arc $y = \sqrt{x}, 1 \leq x \leq 2$ is revolved around the x – axis.The volume of the solid of revolution is _______. a) $\frac{\pi}{2}$ b) $\frac{\pi}{4}$ c) $\frac{3\pi}{2}$ d) $\frac{3\pi}{4}$
CO5	K1	9.	Which of the following is obtained by evaluating $\iint_S \bar{r}.\bar{n}\, ds$ where S is a closed surface and V is the Volume using Gauss Divergence Theorem? a) 3V b) 2V c) V d) 5V
CO5	K2	10.	Which of the following is related with Stoke's Theorem? a) A surface integral and a volume integral b) A Line integral,a surface inegral and a volume integral c) A Line integral and a volume integral d) A Line integral and a surface integral

Course Outcome	Bloom's K-level	Q. No.	**SECTION – B (5 X 5 = 25 Marks)** Answer **ALL** Questions choosing either (a) or (b)
CO1	K3	11a.	If $\bar{A} = u\bar{i} + u^2\bar{j} + u^3\bar{k}$, $\bar{B} = u^3\bar{i} + u^2\bar{j} + u\bar{k}$ find (i) $\frac{d(\bar{A}.\bar{B})}{du}$ (ii) $\frac{d(\bar{A}\times\bar{B})}{du}$ **(OR)**
CO1	K3	11b.	If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ is unit vector, then show that $x\frac{dx}{dt} + y\frac{dy}{dt} + z\frac{dz}{dt} = 0$

CO2	K3	12a.	Find the directional derivative of $\emptyset = x + xy^2 + yz^3$ at (0,1,1) in the direction of the vector $2\bar{\imath} + 2\bar{\jmath} - \bar{k}$
			(OR)
CO2	K3	12b.	Show that $\bar{F} = yz\,\bar{\imath} + zx\,\bar{\jmath} + xy\,\bar{k}$ is irrotational.
CO3	K4	13a.	Prove that $\nabla^2 r^n = n(n+1)r^{n-2}$ where n is a constant.
			(OR)
CO3	K4	13b.	In the vector field $\bar{F} = z(x\bar{\imath} + y\bar{\jmath} + z\bar{k})$, evaluate $\int_C \bar{F}.\bar{r}$ along straight line joining $(0,0,0), (1,1,1)$
CO4	K4	14a.	Evaluate $\iint \bar{A}.\bar{n}\,ds$ if $\bar{A} = 18z\bar{\imath} - 12\bar{\jmath} + 3y\bar{k}$ and S is the surface $2x + 3y + 6z = 12$ in the first octant.
			(OR)
CO4	K4	14b.	Evaluate $\iiint_V \nabla.\bar{F}\,dV$ if $\bar{F} = x^2\bar{\imath} + y^2\bar{\jmath} + z^2\bar{k}$ and if V is the volume of the region enclosed by the curve $0 \le x, y, z \le 1$
CO5	K5	15a.	Show that the volume V of the region enclosed by the surface S is $\frac{1}{3}\iint_S \bar{r}.dS$
			(OR)
CO5	K5	15b.	Using Green's theorem, show that $\int_C (3x + 4y)dx + (2x - 3y)dy = -8\pi$, Where C is the circle $x^2 + y^2 = 4$

Course Outcome	Bloom's K-level	Q. No.	**<u>SECTION – C</u> (5 X 8 = 40 Marks)** **Answer <u>ALL</u> Questions choosing either (a) or (b)**
CO1	K3	16a.	If a, b, w are vector functions of a scalar variable u and if $\frac{da}{du} = w \times a, \frac{db}{du} = w \times b,$ then show that $\frac{d}{du}(a \times b) = w \times (a \times b)$
			(OR)
CO1	K3	16b.	Find the directional derivative of $x + xy^2 + yz^3$ at the point $(0,1,1)$ in the direction whose direction cosines are 2/3,2/3,-1/3.
CO2	K4	17a.	If $\bar{F} = x^2y\bar{\imath} + y^2z\bar{\jmath} + z^2x\bar{k}$, then find curl curl $\bar{F}$
			(OR)
CO2	K4	17b.	Show that the vector $\bar{A} = x^2z^2\bar{\imath} + xyz^2\bar{\jmath} - xz^3\bar{k}$ is solenoidal.
CO3	K4	18a.	Show that $\nabla^2 \log r = \frac{1}{r^2}$.
			(OR)
CO3	K4	18b.	If $\bar{F} = 3xy\bar{\imath} - y^3\bar{\jmath}$, compute $\int_C \bar{F}.\bar{r}$ along $y = 2x^2$ from $(0,0)$ to $(1,2)$.
CO4	K5	19a.	Evaluate the integral $\int_C x\,dx + y\,dy + z\,dz$ where C is the circle $x^2 + y^2 + z^2 = a^2, z = 0$
			(OR)
CO4	K5	19b.	Evaluate $\iiint_V \nabla.\bar{A}\,d\bar{V}$ if $\bar{A} = 2x^2y\bar{\imath} - y^2\bar{\jmath} + 4xz^2\bar{k}$ and V is the region in the first octant bounded by the cylinder $y^2 + z^2 = 9$ and the plane $x = 2$
CO5	K5	20a.	State and prove Stoke's theorem.
			(OR)
CO5	K5	20b.	Using Green's theorem, Evaluate $\int_C x^2(1 + y)\,dx + (y^3 + x^3)dy$ where C is the square formed by the lines $y = \pm 1, x = \pm 1$